

Chapter 9

Anatomy of a GCM

An Atmospheric General Circulation Model (AGCM) is an attempt to solve the equations of motion of the atmosphere and related processes numerically. The aim is to provide simulations that are as comprehensive as practical given the constraints imposed by limited amounts of computer time. By practical here we mean that the model must run fast enough that reasonable simulation results can be obtained within the attention span of scientists working on the problem. In the case of an AGCM with given sea surface temperatures the internal time constants are probably only a few months at most. Then to test the model, compute circulation statistics and do some modest climate change experiments, one needs of the order of a few decades of simulation. The numerical model must be efficient enough to provide such simulations in a month or two of calendar

time.

Describing a recently released AGCM provides a convenient framework for discussing a great number of important issues in climate theory. Examination of the individual components of the AGCM provides the opportunity to discuss in some detail the main processes now thought to be important in the global climate problem. Hence, we embark on a detailed description of the Community Climate Model-Number 2 (CCM2) released in 1994 by the National Center for Atmospheric Research in Boulder, Colorado, USA. We choose this model because it is well documented and widely used in the community. There is considerable debate in the modeling community about how to go about these numerical procedures. Our purpose is to simply use this particular model as an example and a convenient outline of the processes that must be accounted for in such models. Along the way, some opportunity arises to delve slightly into the reasoning behind some of the inevitable choices confronted by a numerical modeling group.

9.1 Introducing CCM2

The CCM2 is a fully three dimensional numerical model of the atmosphere, including moisture tracking, complex cloud interaction, detailed radiative transfer

and the capability for coupling to interactive soil/vegetation and ocean models. Obviously, such a model has to be solved numerically. No numerical model of the atmospheric motions and thermodynamics can fully incorporate every physical mechanism. Many features are simply at scales unresolved by the numerical model. In many cases these so-called subgrid scale effects are treated in some statistical mean or bulk framework. For example, individual clouds cannot be resolved in typical global models. One resorts to relating such features as cloud fraction at a particular altitude level in a grid box to other grid scale variables such as vertical wind speed and grid scale relative humidity. These procedures are known as *parameterizations*. They are inescapable.

To see why fully detailed simulations are impossible, consider a single cubic kilometer of air say in the atmospheric boundary layer. The largest scales of motion are at 1km. There will be important interactions all the way down to scales of 1mm where viscosity dissipates kinetic energy. This is a range of 6 orders of magnitude. If we set up a 3D grid which includes all the relevant scales, we are keeping track of 10^{18} grid points. To conduct the simulation we need to keep track of about 10 variables at the 10^{18} grid points, or 10^{19} numbers. Suppose our computers can operate at 10 gigaflops (10^9 floating point operations per second). Then it takes 10^{10} s to treat all our gridded data by one operation, which is over 3000yr. Hence, brute force simulations which include all relevant spatial scales are out of the question. Parameterization of

smaller scale phenomena in some form is essential.

We can proceed to implement the partial differential equations on a finite difference grid, but problems arise in numerical modeling of a compressible fluid having to do with high frequency phenomena such as sound waves and Lamb waves which are also relatively high frequency, but probably irrelevant to weather and climate. One gets around this problem in atmospheric simulations by a kind of filtering process. The filter process is physically motivated; i.e., it comes from linearized analytical considerations of the equation set. The result of experience suggests that one treat the vertical momentum equation as though the associated acceleration were always zero; in other words the equation is replaced by the barometric equation (local pressure is equal to the weight of gas above). This procedure to a large extent filters out the unwanted high frequency waves allowing larger time steps in the numerical process. Time steps are ordinarily to be smaller than the time for propagation of information across a grid box, otherwise the numerical solutions tend to be unstable, unless special precautions are taken (such as implicit methods, to be discussed later).

In the next few subsections we elaborate how the CCM2 does its job. The treatment here is straight out of the manual written by James J. Hack and his colleagues at NCAR. All references to individual papers on the individual parts can be found in that report.

9.2 Vertical Coordinate

One of the most exasperating problems is the choice of vertical coordinate. Ordinarily one wants a coordinate which has level surfaces coinciding with the fixed material surfaces in the problem in order to cleanly express and enforce the boundary conditions. Since the surface of the planet has topographic features, the radial distance from the center of the Earth is an unsatisfactory coordinate choice. One could choose any variable that is monotonically related to the vertical dimension, z , such as pressure, $p(z)$. The pressure coordinate has the additional advantage of allowing a rather simple expression of the equations of motion, especially if the hydrostatic condition is enforced. Of course, pressure surfaces also intersect the topographical material surface even with level topography since the surface pressure is not uniform at a given instant. A compromise is to use σ coordinates: $\sigma = p/p_s$, where p_s is the surface pressure. Note that $0 \leq \sigma \leq 1$. This is an example of a terrain-following coordinate. Earlier versions of CCM used σ coordinates, but the CCM2 employs a slight generalization of it, the η coordinate. This is a transformation from the vertical z coordinate to one which is monotonic in pressure p and depends on the surface pressure, denoted by p_s . The coordinate $\eta(p, p_s)$ should be such that

1. $\eta(\pi, p_s) = 1$

2. $\eta(0, p_s) = 0$
3. $\eta(p_{top}, p_s) = \eta_{top}$ where p_{top} denotes the pressure at the top of the model.

The σ coordinate, $\sigma = p/\pi$ is a special case of an η coordinate. The η coordinate has no intersections of its level surfaces with the surface of the planet. In numerical modeling it is known that when such surfaces intersect, there is a large local numerical error made at each time step. To close the system, one needs boundary conditions that allow no mass to enter from top or bottom of the atmosphere: $\dot{\eta}(p_s, p_s) = 0$ and $\dot{\eta}(p_{top}, p_s) = 0$.

9.2.1 Governing Equations

Given these definitions and the physics of the problem, prognostic equations can be written for:

$$\frac{\partial \zeta}{\partial t} = \mathbf{k} \cdot \nabla \times (\mathbf{n} / \cos \phi) + F_{\zeta_H} \quad (9.1)$$

$$\frac{\partial \zeta}{\partial t} = \nabla \cdot (\mathbf{n} / \cos \phi) - \nabla^2 (E + \Phi) + F_{\delta_H} \quad (9.2)$$

$$\frac{\partial T}{\partial t} = \frac{-1}{a \cos^2 \phi} \left[\frac{\partial}{\partial \lambda} (UT) + \cos \phi \frac{\partial}{\partial \phi} (VT) \right] + T\delta - \dot{\eta} \frac{\partial T}{\partial \eta} + \frac{R}{c_p^*} \mathcal{T} \frac{\omega}{p} \quad (9.3)$$

$$+ Q + F_{T_H} + F_{F_H} \quad (9.4)$$

$$\frac{\partial q}{\partial t} = \frac{-1}{a \cos^2 \phi} \left[\frac{\partial}{\partial \lambda} (Uq) + \cos \phi \frac{\partial}{\partial \phi} (Vq) \right] + q\delta - \dot{\eta} \frac{\partial q}{\partial \eta} + S \quad (9.5)$$

$$\frac{\partial p_s}{\partial t} = - \int_1^{\eta_t} \nabla \cdot \left(\frac{\partial p}{\partial \eta} \mathbf{V} \right) d\eta \quad (9.6)$$

Here the virtual temperature is denoted by \mathcal{T} and with $\mathbf{n} = (n_U, n_V)$:

$$n_U = +(\zeta + f)V - \dot{\eta} \frac{\partial U}{\partial \eta} - R \frac{\mathcal{T}}{p} \frac{1}{a} \frac{\partial p}{\partial \lambda} + F_U \quad (9.7)$$

$$n_V = -(\zeta + f)V - \dot{\eta} \frac{\partial V}{\partial \eta} - R \frac{\mathcal{T}}{p} \frac{\cos \phi}{a} \frac{\partial p}{\partial \phi} + F_V \quad (9.8)$$

$$E = \frac{U^2 + V^2}{2 \cos^2 \phi} \quad (9.9)$$

$$(U, V) = (u, v) \cos \phi \quad (9.10)$$

$$\mathcal{T} = \left[1 + \left(\frac{R_v}{R} - 1 \right) q \right] T \quad (9.11)$$

$$c_p^* = \left[1 + \left(\frac{c_{pv}}{c_p} - 1 \right) q \right] c_p \quad (9.12)$$

In addition to the these there are three diagnostic equations:

$$\Phi = \Phi_s + R \int_{p(\eta)}^{p(1)} \mathcal{T} d \ln p \quad (9.13)$$

$$\dot{\eta} \frac{\partial p}{\partial \eta} = -\frac{\partial p}{\partial t} - \int_{\eta_t}^{\eta} \nabla \cdot \left(\frac{\partial p}{\partial \eta} \mathbf{V} \right) d\eta \quad (9.14)$$

$$\omega = \mathbf{V} \cdot \nabla p - \int_{\eta_t}^{\eta} \nabla \cdot \left(\frac{\partial p}{\partial \eta} \mathbf{V} \right) d\eta \quad (9.15)$$

The equations described above are finite differenced on 18 levels in η . Roughly speaking, the finite vertical difference increments are equal in atmospheric mass. The horizontal numerical algorithm on constant η surfaces is accomplished spectrally using spherical harmonics as a basis set.

9.3 Horizontal Representation

The vorticity, divergence and $U = u \cos \theta, V = v \cos \theta$ are expanded into spherical harmonics. Some of the dynamics can then be done analytically using $\nabla^{2n} Y_{lm} = (-l(l+1))^n Y_{lm}$. In addition to this increase in efficiency some quadratic invariants such as energy and vorticity are preserved for a given level of truncation (as shown in the last chapter). Here the level of truncation becomes important. In the CCM2 the level of truncation is specifiable. It is widely used at truncation T42, but in the illustrations shown in this book we use R15. Intercomparisons between these two suggest that the coarser level is sufficient for what we are trying to illustrate in this chapter.

The fields must be continually transformed back and forth between spectral representation and a horizontal physical grid, the latter being equally separated points along longitude and Gaussian integration points along latitude. The order of these depends on the level of spectral truncation. The integrals of products of spherical harmonics and a field amplitude around latitude circles can be accomplished by Fast Fourier Transform. Those along meridians can be done with the N point Gaussian quadrature.

In addition to the specifications above there needs to be some form of horizontal diffusion in the model. This is an attempt to imitate the subgrid scale effects which are not otherwise accounted for in the finite difference (or spectral) formulation. The diffusion is generally of the form $\nabla^2(\cdot)$ or $\nabla^4(\cdot)$ where (\cdot) might be any one of the fields. The strength of these terms is generally tuned to give reasonable kinetic energy spectra, etc.

9.4 Time Stepping

The prognostic equations are to be time stepped by some finite difference procedure. This process is very complicated in CCM2. In general a prognostic

equation might be represented as:

$$\frac{\partial\psi}{\partial t} = P_T(\psi) + \Gamma(\psi) + F(\psi) + P_A(\psi) \quad (9.16)$$

where P_T represents the usual right hand sides of prognostic equations represented as tendencies including heating rates, drag forcings, cloud parameterizations, etc.; P_A represents those terms applied as adjustments (such as the convective adjustment illustrated in Chapter 4, though not in that crude form), Γ represents the dynamical components (advection, coriolis, etc.) and the term F is an ad hoc correction to ensure conservation of atmospheric mass and water vapor by the dynamical processes. Completing one time step involves a mixture of treatments for each of the terms on the right hand side above. Assume all variables ψ^n are known at time n on the Gaussian grid and time filtered values $\bar{\psi}^{n-1}$ are known at $n - 1$. After all variables are updated on the grid by $n + 1$ the time step is complete. To give a crude idea of how this happens define

$$\psi^- = \bar{\psi}^{n-1} + 2\Delta t P_T^n(\psi^-, \psi^n, \bar{\psi}^{n-1}) \quad (9.17)$$

Note that ψ^- also appears on the RHS so that this procedure is an implicit one; i.e., the equation has to be *solved* for ψ^- . This is accomplished by linearizing about the present state. Next a quantity $\hat{\psi}^+$ is computed by adding to ψ^- the tendency due to dynamics and advection of water substance, again an implicit procedure. Next the adjustments are applied and the corrections for global

mass. these steps essentially produce ψ^{n+1} . Finally, the filter is applied to give $\overline{\psi}^n$:

$$\overline{\psi}^n = \psi^n + \alpha \left(\overline{\psi}^{n-1} - 2n\psi^n + \psi^{n+1} \right) \quad (9.18)$$

9.5 Clouds

The CCM2 attempts to compute the fraction of a grid box covered by cloud at in three vertical zones: p_s to 700mb, 700mb to 400mb and 400mb to the model top. The clouds are considered to be randomly overlapping and can form at any of the model levels except near the ground. Low level frontal clouds can occur for any value of vertical air speed and middle and upper level cloud formation thresholds are allowed to depend on relative humidity and stability. Cloud types include convective and stratus. The actual cloudiness fraction is computed by rather simple semi-empirical formulas with a large number of adjusted coefficients.

9.6 Radiation

In order to study its effects on latent heating and the land surface interaction, a diurnal cycle has been included in CCM2.

9.6.1 Solar Radiation

The interaction of solar radiation is calculated by means of the δ -Eddington approximation which was introduced in Chapter 3; for details see the paper by Briegleb (1992). For molecular scattering and absorption (O_3 , CO_2 , O_2 , and H_2O) the spectrum is broken into 18 intervals in which the parameters of the δ -Eddington scheme are required. Comparison of the vertical profiles of computed heating rates with more exact calculations using very narrow bands (5 cm^{-1}) has been shown to be good.

When clouds are encountered, the cloud extinction optical depth is determined by the liquid water pathlength and the effective cloud drop radius. The single particle backscatter coefficients and asymmetry parameters are given in terms of parametric expressions over four spectral ranges, each depending on effective average drop size. The effective optical depth for randomly overlapping clouds is handled by a simple parameterization.

The solar radiation scheme is evaluated once every model hour over the sunlit portions of the earth. Surface albedos are given in two bands only, visible and infrared, with a distinction between direct and diffuse radiation. The distribution of the amount of cloud is calculated every hour just before the solar radiation is computed.

The several constituents encountered in a layer are accounted for by effective band values (Cess, 1985):

$$\tau = \sum_i \tau_i \quad (9.19)$$

$$\varpi = \frac{\sum_i \varpi_i \tau_i}{\tau} \quad (9.20)$$

$$g = \frac{\sum_i g_i \varpi_i \tau_i}{\varpi \tau} \quad (9.21)$$

$$f = \frac{\sum_i f_i \varpi_i \tau_i}{\varpi \tau} \quad (9.22)$$

where the notation is the same as in Chapter 3. Once the analytical solutions for a given layer are evaluated going up and down the atmospheric column, we can give the following bulk quantities for the layer (following Briegleb, 1992):

$e^{-\tau/\mu_0}$ = direct beam transmission from top of the atmosphere to the interface
(here τ is the effective scaled value of τ used in the δ -Eddington solution)

and it is the scaled optical depth from the top-of-the-atmosphere to the interface).

$R^\uparrow(\mu_0)$ = reflectivity to direct solar radiation of entire atmosphere below the surface.

$T^\downarrow(\mu_0)$ = total transmission to direct solar radiation incident from above to entire atmosphere above the interface.

\bar{R}^\uparrow = reflectivity of atmosphere below the interface to diffuse radiation from above.

\bar{R}^\downarrow = reflectivity of atmosphere above the interface to diffuse radiation from below.

Once these quantities are computed for an interface the upward and downward fluxes of shortwave radiation can be computed:

$$F_{sw}^\uparrow = \frac{e^{-\tau/\mu_0} R^\uparrow(\mu_0) + (T^\downarrow(\mu_0) - e^{-\tau/\mu_0}) \bar{R}^\uparrow}{1 - \bar{R}^\downarrow \bar{R}^\uparrow} \quad (9.23)$$

$$F_{sw}^\downarrow = e^{-\tau/m\mu_0} + \frac{(T^\downarrow(\mu_0) - e^{-\tau/\mu_0}) + e^{-\tau/\mu_0} R^\uparrow(\mu_0) \bar{R}^\downarrow}{1 - \bar{R}^\downarrow \bar{R}^\uparrow} \quad (9.24)$$

Now the flux contributions from the different bands can be added up to give

the total interface fluxes up and down. Once these are known the divergences can be computed to arrive at the layer heating rates.

In going up and down the atmospheric column one encounters the surface. This means we need to include the albedo of the local surface. In CCM2 the Earth's surface is partitioned into $1^\circ \times 1^\circ$ boxes and a separate albedo is specified for each box. In addition the albedo of a surface element falls into two zenith angle dependence classes: strong to weak. The strong class gets a multiplicative factor $r_s = 1.4/(1 + 0.4\mu_0)$ and the weak, $r_w = 1.1/(1 + 0.2\mu_0)$, where μ_0 is the instantaneous solar zenith angle. Table ?? shows the values used for the 12 different surface types included in CCM2.

9.6.2 Longwave Radiation

A GCM must take into account the wavelength dependence of absorptivity of infrared radiation due to atmospheric constituents. Fluxes up and down, $F_{ir}^\uparrow(p_k)$ and $F_{ir}^\downarrow(p_k)$ are computed at each pressure level p_k , where the upward and downward fluxes are defined as the integrals over the upward and downward hemispheres as in Chapter 3. The layer-effective absorptivity over a given

Table 9.1: Surface albedos (%) for visible (0.2-0.7 μm) and near-infrared (0.7-5.0 μm) for various surface types used in CCM2. Also shown is the broadband (bb) albedo although it is not actually used in CCM2 calculations. The strength of zenith angle dependence for the different types is denoted by zen type.

Type	Description	vis	nir	bb	zen type
0	Ocean	6	6	6	strong
1	Mixed farming tall grass	9	28	19	strong
2A	Tall grassland	11	33	23	strong
2B	Short grassland	14	34	25	strong
3	Evergreen forest	6	22	14	weak
4	Mixed deciduous/evergreen	6	27	17	weak
5	Deciduous	6	31	19	weak
6	Tropical forest	6	22	14	weak
7	Med tall grass/woodlands	6	28	18	weak
8A	Desert, sandy	35	51	43	strong
8B	Desert, rocky	24	40	32	strong
9	Tundra	10	27	19	strong
10	Land/sea ice	90	65	77	weak
11	Snow	95	70	82	

spectral wavenumber band is given by

$$\alpha(p, p') = \frac{1}{\frac{dB}{dT}(p')} \int A_{\tilde{\nu}}(p, p') \frac{dB_{\tilde{\nu}}}{dT}(p') d\tilde{\nu} \quad (9.25)$$

where $A_{\tilde{\nu}}$ is the local absorptivity at spectral wavenumber $\tilde{\nu} (\equiv 1/\text{wavelength})$ and $B_{\tilde{\nu}}(p)$ is the black body radiation formula evaluated at pressure level p , and $B(p) = \sigma T^4(p)$. The emissivity of the layer is

$$\epsilon(0, p) = \frac{1}{B(0)} \int A_{\tilde{\nu}}(0, p) B_{\tilde{\nu}}(0) d\tilde{\nu} \quad (9.26)$$

As before these can be used to compute the fluxes for clear media:

$$F_{clr}^{\downarrow}(p_k) = B(0)\epsilon(0, p_k) + \int_0^{p_k} \alpha(p', p_k) \frac{dB}{dp'}(p') dp' \quad (9.27)$$

$$F_{clr}^{\uparrow}(p_k) = \sigma T_S^4 - \int_{p_k}^{p_s} \alpha(p', p_k) \frac{dB}{dp'}(p') dp' \quad (9.28)$$

In other words, each atmospheric layer is assumed to absorb and emit as a gray slab with an effective emissivity calculated by averaging over several infrared bands in wavenumber. Water vapor has to be treated separately since it has absorption bands extending throughout the entire longwave range.

There are three main contributors to the infrared absorption/emission in the clear portion of the atmospheric column: CO_2 , H_2O , and O_3 . In CCM2

their contributions are additive in $\alpha(p, p')$. Also taken into account are the (parameterized) pressure and temperature dependences of the absorptivities in each layer.

Longwave interaction with cloud is estimated by taking cloud surfaces to be essentially black in the longwave, turning gray as a function of lessening liquid water path length according to $\epsilon(p_k) = 1 - \exp(-.1LWP(k))$, where $\epsilon(p_k)$ is the broad band emissivity of the cloud at layer p_k , and $LWP(k)$ is the cloud liquid water path length; the $0.1 \text{ m}^2\text{g}^{-1}$ factor is an absorption coefficient based on observations. At each model level an effective cloud amount for each model layer is defined:

$$A'_k = \epsilon(p_k)A_k \quad (9.29)$$

A probability of a cloud existing in layer k , and clear sky below this layer is given by

$$f_{cld}(k) = A'_k \prod_{i=2}^{k-1} (1 - A'_i) \quad (9.30)$$

The clear sky fraction for the column is

$$f_{clear} = \prod_{i=1}^N (1 - A'_i) \quad (9.31)$$

The fluxes are evaluated at each model half-layer for longwave heating-rate calculations on full pressure-levels.

Computing the upward and downward fluxes of longwave radiation is rather expensive, since an iterative finite difference algorithm going up and down the atmospheric column must be solved to compute these fluxes. Over 90% of the computation time is spent on computing the α and ϵ at the various levels. To save time these are only recomputed at about 12 hr intervals (user adjustable). The longwave heating rates are computed more frequently: typically at intervals of one hour.

9.6.3 Surface Energy Exchanges

The net flux of energy from the surface is given by

$$F_{net}(T_s) = F_{RAD} - \sigma T_s^4 - \mathcal{H} - \mathcal{L} \quad (9.32)$$

where \mathcal{H} represents the net sensible heat flux and \mathcal{L} represents the net latent heat flux (evaporation/condensation). These fluxes have to be parameterized in some bulk form. For example,

$$\mathcal{H} = c_p \rho_1 C_H |\mathbf{V}_1| (\theta_s - \theta_1) \quad (9.33)$$

where c_p is the heat capacity of air at constant pressure; ρ_1 is the air density at level 1 in the model; \mathbf{V}_1 is the horizontal velocity at level 1; θ_s and θ_1 are the potential temperature at the surface and level 1. The coefficient C_H is far

more involved. It comes from scaling arguments in boundary layer physics. It is composed of two factors:

$$C_{\mathcal{H}} = C_N f_{\mathcal{H}}(Ri_s) \quad (9.34)$$

where the first factor is the neutral exchange coefficient (neutral referring to atmospheric stability)

$$C_N = \frac{k^2}{\ln((z_1 + z_{0\mathcal{M}})/z_{0\mathcal{M}}) \ln((z_1 + z_{0\mathcal{H}})/z_{0\mathcal{H}})} \quad (9.35)$$

where k is the von Karman constant (≈ 0.4), z_1 is the height of level one, and $z_{0\mathcal{M}}$ is the roughness length for momentum and $z_{0\mathcal{H}}$ that for heat; this latter can be specified to allow for different types of land cover.

The function $f_H(Ri_s)$ has to come from observations or a detailed theory of boundary layer turbulence (which does not exist). The variable Ri_s is the Richardson number,

$$Ri_s = \frac{gz_1(\theta_{v1} - \theta_{vs})}{\theta_1 |\mathbf{V}_1|^2} \quad (9.36)$$

a dimensionless measure of the degree of atmospheric stability. In the formula, θ_v refers to virtual potential temperature at the indicated levels. The Richardson number is negative for unstable conditions. Under unstable conditions the

model uses the empirical form

$$f_H(Ri) = 1 - \frac{15Ri}{1 + 75C_N\{((z_1 + z_0)/z_0)|Ri|\}^{1/2}} \quad (9.37)$$

and for stable conditions, $Ri_s \geq 0$,

$$f_H(Ri) = \frac{1}{1 + 10Ri(1 + 8Ri)} \quad (9.38)$$

The formulation for the latent heat term parallels that for sensible heat:

$$L = L_0\rho_1 D_w C_H |\mathbf{V}_1| (q_s^* - q_1) \quad (9.39)$$

where $L - 0$ is the latent heat associated with a unit mass of water being condensed, q_1 is the specific humidity at level one, and q_s^* is the saturation vapor pressure at the local temperature. The factor D_w is a wetness factor which in principle, might be computed for a specific vegetative (or other) cover.

Momentum transfer at the surface interface (drag) is formulated in a very similar way in CCM2 with semi-empirical functions like $f_M(Ri_s)$, etc.

9.7 Surface/Soil Temperature

The subsurface over land or sea ice is a heat conductor whose local temperature obeys a linear heat conduction equation.

$$\rho C \frac{\partial T}{\partial t} = \frac{\partial}{\partial z} \left(\kappa \frac{\partial T}{\partial z} \right) \quad (9.40)$$

The model allows horizontally variable thermal conductivity, etc, from one horizontal grid box to another. The heat diffusion equation is split vertically into layers (specifiable but typically 4):

$$\rho_\ell C_\ell \left(\frac{T_\ell^{n+1} - T_\ell^n}{\Delta t} \right) = \left(\frac{\partial}{\partial z} \left(\kappa \frac{\partial T}{\partial z} \right) \right)_\ell^{n+1} \quad (9.41)$$

and is solved iteratively to match fluxes at the surface interface.

The net surface energy flux is given by

$$F_{\text{net}}(T_s) = F_{\text{RAD}} - \sigma T_s^4 - c_p \rho_1 (\overline{w'\theta'})_s - L(\overline{w'q'})_s \quad (9.42)$$

9.8 Vertical Diffusion

Because of the way the vertical momentum equation is handled to filter high frequency motions, the momentum exchange between ‘horizontal’ layers is mod-

eled by a diffusion (macroviscosity) process. For example, the horizontal winds at a particular level will experience a stress due to eddy exchanges with adjacent layers. This is parameterized by diffusion:

$$\frac{\partial u}{\partial t} = -\frac{1}{\rho} \frac{\partial \tau_\lambda}{\partial p} \quad (9.43)$$

where τ_λ is the appropriate component of the stress tensor which is modeled by

$$\tau_\lambda = -\rho K_m \frac{\partial u}{\partial z} = g \rho^2 K_c \frac{\partial u}{\partial p} \quad (9.44)$$

similar forms hold for the v component and for vertical heat moisture transfer. The variable K_c (c can be different for the different quantities being diffused) is the so-called ‘eddy-diffusivity’ which is a function of ℓ_c , a length scale, and local vertical gradients of wind and virtual potential temperature. For example,

$$K_c = \ell_c^2 \left| \frac{\partial \mathbf{V}}{\partial z} \right| F_c(Ri) \quad (9.45)$$

The mixing length ℓ_c is given by

$$\ell_c = \frac{1}{kz} + \frac{1}{30\text{meters}} \quad (9.46)$$

where again k is the Von Karman constant. Finally,

$$Ri = \frac{g}{\theta_v} \frac{\partial \theta_v / \partial z}{\left| \frac{\partial \mathbf{V}}{\partial z} \right|^2} \quad (9.47)$$

is the gradient Richardson Number. It is a dimensionless parameter measuring the ratio of generation of buoyant kinetic energy to mechanically generated turbulent kinetic energy. The virtual potential temperature

$$\theta_v = \theta [(1 + 0.61)q] \quad (9.48)$$

is the temperature at which dry air has the same density as moist air at the same pressure. Its purpose here is to simply account for the density (and therefore buoyancy) difference between moist and dry air. For the function F_c , under unstable conditions CCM2 uses

$$F_c(Ri) = (1 - 18Ri)^{1/2} \quad (9.49)$$

For stable conditions ($Ri > 0$) it uses (9.38). In this last case, there is no distinction between vertical diffusion of heat, scalars and momentum.

9.9 Atmospheric Boundary Layer

The free atmosphere is that part which flows without the influence of ‘friction’ from below; i.e., where near-geostrophic conditions can exist. Somewhere between a few and one km above the surface a rather distinct interface between the free atmosphere and a well mixed layer below occurs, the region below this interface is the so-called *atmospheric boundary layer* (ABL). In the lowest

few meters to about one tenth of the height of the ABL another interface occurs; below it is the *surface boundary layer* (SBL). The atmospheric motion is making the transition from the near-geostrophic conditions above the ABL to near zero speeds at the ground. Over short adjustment intervals vertical gradients can be large, with the shear generating mechanical turbulence. This turbulence causes a nearly uniform mixing of the properties horizontal velocity, temperature and moisture over the region between the SBL and the top of the ABL; this intermediate zone is called the *mixed layer*. In the mixed layer the ratio z/h , where h is the ABL thickness, is a good dimensionless variable with which to express various turbulent quantities such as kinetic energy, vertical fluxes, etc. Several functions have a universal form when expressed as functions of this variable (no matter what the value of h). Much of the goal of boundary layer theory is to search out and refine such scaling variables.

The ABL passes through a cycle of states driven by the diurnal cycle. At night h is basically small ($0 < h < 250\text{m}$) representing a stable inversion layer near the surface. At sunrise the ABL thickness zooms up to the order of a km or two above which there may be a capping inversion. At sunset the capping inversion erodes into possibly several transient weak inversion layers. In the tropics, the situation is slightly different. The trade wind inversion which is nearly always present (at roughly 1.5 km) tends to serve as the top of the ABL.

During the daytime the ABL is characterized by large convective plumes stretching from the surface all the way to the its top. These overturning eddies tend to have horizontal as well as vertical length scales roughly the size of the ABL thickness. Here is the mechanism for rapid homogenization of the mixed layer.

In the SBL, z/h is not such a good dimensionless variable (i.e., universal functions for eddy-generated fluxes do not seem to be unique when expressed as a function of it). A much better choice is z/L where L is the Monin-Obhukov length scale. It is defined through

$$\frac{z}{L} = -\frac{g}{\bar{\theta}} \frac{\mathcal{H}}{u_*^3/kz} \quad (9.50)$$

where \mathcal{H} is the eddy generated heat flux at the surface, u_* is the so-called friction velocity. This last is closely related to the stress (drag) exerted by the surface on the horizontally flowing air just above it:

$$\tau_\lambda = \rho u_*^2 \quad (9.51)$$

The expression for z/L is very similar to the defining espression for the flux Richardson number

$$Ri_f = \frac{g}{\bar{\theta}} \frac{\mathcal{H}}{\mathcal{M}(\partial u/\partial z)} \quad (9.52)$$

where \mathcal{H} is the local eddy-generated heat flux and \mathcal{M} is the local eddy-generated momentum flux.

In the CCM2 an attempt is made to account for the large scale overturning found in the ABL under convective conditions. For example, heat fluxes are computed by

$$\mathcal{H} = -K_c \left(\frac{\partial \theta}{\partial z} - \gamma_c \right) \quad (9.53)$$

and

$$K_c = kw_t z \left(1 - \frac{z}{h} \right)^2 \quad (9.54)$$

where w_t is a velocity scale and γ_c is a “non-local” transport term reflecting the transport due to dry convection. Above the ABL, $\gamma_c = 0$ so that the transport form is the same as that above. Inside the ABL estimation of γ_c involves use of some quantities which have to come from an estimation of h . The subscript c means that it depends upon which quantity being convected (heat, etc.).

A crucial step in the process is to estimate the ABL height h . In CCM2 this is done by iteratively solving

$$h = \frac{Ri_{cr} \{u(h)^2 + v(h)^2\}^2}{(g/\theta_s)(\theta_v(h) - \theta_s)} \quad (9.55)$$

where Ri_{cr} is a critical value of Richardson number taken to be 0.5 in the model. Once the value of h is found, the velocity scales, and γ_c can be calculated.

Finally, we are in position to compute vertical diffusion using

$$\frac{\partial C}{\partial t} = \frac{1}{\rho} \frac{\partial}{\partial z} \left[\rho K_c \left(\frac{\partial C}{\partial z} - \gamma_c \right) \right] \quad (9.56)$$

The time differencing is by a split implicit method. Since C does not occur in the second term and explicit method is used for this term, but an implicit scheme is used for the first term.

At this point vertical diffusion is solved for the entire vertical column for the horizontal velocity components, the potential temperature, and the water vapor.

9.10 Cumulus Parametrization

The equatorial tropics is characterized by convergence in the atmospheric boundary layer into a latitudinally narrow strip of convectively driven rising air. The belt of converge migrates Northward in Northern summer and recrosses the Equator after Equinox. As the moisture laden air converges in the boundary layer it rises and converts latent heat beginning the condensation level into sensible heat and therefore more buoyancy. This process is started by the natural convective rising of dry air at the latitude of maximal solar heating. Latent heat release accelerates the convective overturning, converting to kinetic energy

the latent heat gathered in evaporation along the surface in the Trade Wind zones. In the *Intertropical Convergence Zone* (ITCZ) the convection is very deep, penetrating in individual cloud towers to the tropical tropopause. The horizontal scales of the vertical motion are only a few km. The importance of somehow including the moist tropical convection in an AGCM cannot be overemphasized, since the tropical circulation has so much to do with the extratropical circulation and the distributions of moisture in the tropical upper troposphere are determined by this circulation. Clearly, the vertical profile of water vapor and cloud in the tropics, which cover about 50% of the Earth, will have a large impact on radiation feedbacks in numerical climate change experiments.

The question here is how can this convective process be included in AGCMs? The convection occurs on spatial scales that are small compared to the resolution of most feasible implementations of the AGCMs. We may be fortunate compared to some subgrid scale parameterization problems in for example turbulence. In the case of turbulence energy is distributed over a continuum of scales spanning the range from well above gridbox size down to millimeters. In our case the phenomenon we are trying to describe is confined to horizontally very small scales well below the gridbox size leading essentially to a spectral gap. One would like a parametrization that renders correct values of dynamical and thermodynamical variables at the AGCM gridbox scales based upon inputs

that are also at the model grid scale (typically a few hundred km). The main variables that might be delivered are the mass flux of air in the convection, the flux of water substance and the flux of momentum. The first issue that arises is the very possibility of such a scheme.

Observational studies by many workers since the late 1950's (e.g., Riehl and Malkus, 1958, up to Krishnamurti et al., 1980, 1983) have supported the idea that there is a plausible connection between the values of large scale thermodynamic and flow fields which can be used as input to the parameterization of subgrid scale effects. Theoretically, for such a scheme to work, we must imagine that for a given grid box there are many small scale events quasi-randomly occurring over a time interval roughly equal to the time step in the model. If this is the case a kind of quasi-equilibrium might exist between the statistics of the events at the subgrid scale level and the external larger scale environment. If this is the case we can use ensemble statistics to describe the action of the many statistically independent events inside the box. Most convection schemes rely on this kind of thinking.

9.10.1 Subgrid-scale formulations

There are many different formulations of moist convection schemes currently being implemented in AGCMs. It seems that the choice at this time may depend more on style and taste than upon reason followed by rigorous hypothesis testing in the field, since the tests are so difficult to carry out and the need for a working scheme is so urgent in AGCM construction. Mention is made here of two schemes besides the one used in CCM2 (all of these are thoroughly discussed in the books by Cotton and Anthes, 1989, and Emanuel, 1993, and in the article by Tiedke, 1988). The starting point is a set of equations:

$$\frac{\partial T}{\partial t} + \nabla \cdot \mathbf{V}T + \frac{\partial \omega T}{\partial p} - \frac{\omega RT}{c_p p} = \frac{L}{c_p} C^* + Q_R \quad (9.57)$$

$$\frac{\partial r_v}{\partial t} + \nabla \cdot \mathbf{V}r_v + \frac{\partial \omega r_v}{\partial p} = -C^* \quad (9.58)$$

$$\frac{\partial \zeta}{\partial t} + \nabla \cdot \mathbf{V}\zeta_a + \omega \frac{\partial \zeta}{\partial p} + \hat{\mathbf{k}} \cdot \nabla \omega \times \frac{\partial \mathbf{V}}{\partial p} = 0 \quad (9.59)$$

where T is temperature, \mathbf{V} is the horizontal velocity, ω is the vertical velocity in pressure coordinates (dp/dt), R is the gas constant, L is the latent heat of condensation, c_p the heat capacity at constant pressure, Q_R is the heating rate due to radiation effects, r_v is the water vapor mixing ratio, C^* is the net

condensation rate, ζ is the vertical component of relative vorticity, $\zeta_a = \zeta + f$ and f is the Coriolis parameter.

The variables in the last 3 equations are understood to apply to individual parcels of air much smaller than the size of a AGCM grid box. We can apply a *Reynolds* averaging procedure (it's really an ensemble average) by separating the box average value (denoted by the overbar) from its departure from the box average (denoted by primes):

$$r_v = \bar{r}_v + r'_v \quad (9.60)$$

$$T = \bar{T} + T' \quad (9.61)$$

$$\zeta = \bar{\zeta} + \zeta' \quad (9.62)$$

After writing the equations in terms of these quantities and then taking a Reynolds average on the system we obtain:

$$\frac{\partial \bar{r}_v}{\partial t} + \nabla \cdot \bar{\mathbf{V}} \bar{r}_v + \frac{\partial \bar{\omega} \bar{r}_v}{\partial p} = -\bar{C}^* - \nabla \cdot \overline{\mathbf{V}' r'_v} - \frac{\partial \overline{\omega' r'_v}}{\partial p} \quad (9.63)$$

$$\frac{\partial \bar{T}}{\partial t} + \nabla \cdot \bar{\mathbf{V}} \bar{T} + \frac{\partial \bar{\omega} \bar{T}}{\partial p} - \frac{\bar{\omega} \bar{\alpha}}{c_p} = \frac{L}{c_p} \bar{C}^* + \bar{Q}_R - \nabla \cdot \overline{\mathbf{V}' T'} - \frac{\partial \overline{\omega' T'}}{\partial p} - \frac{\overline{\omega' \alpha'}}{c_p} \quad (9.64)$$

where α is specific volume ($\alpha = RT/p$),

$$\frac{\partial \bar{\zeta}}{\partial t} + \nabla \cdot \bar{\mathbf{V}} \bar{\zeta} + \bar{\omega} \frac{\partial \bar{\zeta}}{\partial p} + \hat{k} \cdot \nabla \bar{\omega} \times \frac{\partial \bar{\mathbf{V}}}{\partial p} = -\nabla \cdot \overline{\mathbf{V}' \zeta'_a} - \overline{\omega' \frac{\partial \zeta'}{\partial p}} - \hat{k} \cdot \overline{\nabla \omega' \times \frac{\partial \mathbf{V}'}{\partial p}} \quad (9.65)$$

Similarly equations for the potential temperature θ , the dry static energy $s = c_p T + gz$ or the moist static energy $h = c_p T + gz + Lr_v = s + Lr_v$ can be derived and these are used as starting points in some of the formulations. For example,

$$\frac{\partial s}{\partial t} + \nabla \cdot \mathbf{V}s + \frac{\partial \omega s}{\partial p} = LC^* + c_p Q_R - \frac{\partial \overline{\omega' s'}}{\partial p} \quad (9.66)$$

A typical approach is to divide the area into cumulus cloud (quantities denoted by subscript c) and noncloud areas (quantities denoted with tilde). The areal fraction occupied by cumulus is a . The mass flux of rising air is

$$\overline{M} = -\bar{\omega} \quad (9.67)$$

$$= -a\omega_c - (1-a)\tilde{\omega} \quad (9.68)$$

$$= M_c + \tilde{M} \quad (9.69)$$

Ordinarily we might expect M_c to be positive (rising air in the cumulus updrafts) and \tilde{M} to be negative indicating subsidence; however, there could be overall lifting throughout the grid box. The dry static energy and the water vapor can be subdivided:

$$s = as_c + (1-a)\tilde{s} \quad (9.70)$$

$$r_v = ar_{vc} + (1-a)\tilde{r}_v \quad (9.71)$$

These can be used along with the conditions $a \ll 1$, $|\omega_c| \gg |\tilde{\omega}|$ to derive

$$\overline{\omega' s'} \approx a\omega_c(s_c - \tilde{s}) = -M_c(s_c - \tilde{s}) \quad (9.72)$$

$$\overline{\omega' r'_v} \approx a \omega_c (r_{vc} - \tilde{s}) = -M_c (r_{vc} - \tilde{r}_v) \quad (9.73)$$

These allow us to close the equations for s and r_v by substituting for the Reynolds quantities $\overline{\omega' s'}$ and $\overline{\omega' r'_v}$. Finally, we must average over all the cloud types in the ensemble or grid box. Each is assumed to have area a_i and $\sum_i a_i \ll 1$. These sums are inserted into the governing equations. The procedure from here is to find simple relationships between the various parameters which have been introduced. These can be tested against observed quantities during field experiments. A number of these are described elsewhere; the Kuo-Anthes schemes are presented by Cotton and Anthes, 1989; the Arakawa-Shubert schemes are especially well documented in Tiedke, 1988, and good philosophical discussions are given by Emanuel, 1993.

9.10.2 Moist convective adjustment

This scheme is similar to the convective adjustment introduced in Chap. 3 in our radiative convective models. In the simplest of these formulations the vertical profile of equivalent potential temperature θ_e is examined at each AGCM time step. If the derivative $\partial\theta_e/\partial p > 0$ (convectively unstable), the profile is immediately adjusted until the profile is constant. This is accomplished by iterating on the moist static energy equation, the saturation vapor pressure's dependence on temperature, the hydrostatic equation and the relation between

the saturation vapor pressure and the mixing ratio. Essentially the solution is adjusted onto a moist adiabat. In the process there may be precipitation. This procedure is the so-called hard adjustment. It produces too little precipitation according to a comparison with observations (Krishnamurti et al., 1980).

The so-called soft adjustment assumes the adjustment only occurs over the cumulus cloud cross-sectional area a , the remaining area $(1 - a)$ remaining at its initial vertical profile. The final values of temperature and water vapor are given by

$$T^{final} = aT_c + (1 - a)T^{initial} \quad (9.74)$$

$$r_v^{final} = ar_{vc} + (1 - a)r_v^{initial} \quad (9.75)$$

where T_c and r_{vc} are determined by a wet adiabat. The value of a is chosen to maintain a fixed value of the average relative humidity, taken to be about 80%, a value found to fit the observations best (Krishnamurti et al., 1980). The soft adjustment schemes are used in the GFDL AGCM (Manabe et al., 1965)

9.10.3 Hack's mass flux scheme

In keeping with the philosophy of this chapter we give in some detail the approach taken in CCM2. This scheme was developed by Hack (1994). Its starting point are equations similar to (9.66) and (9.63) except that the budget of liquid

water l (g/kg) is retained in the cumulus cloud areas. Liquid water substance is added to r_v , $q_{v+l} = r_v + l$, and subtracted from the dry static energy in the form $s_l = s - Ll$. The latter is the static energy analog to equivalent potential temperature (Betts, 1975). The liquid water l is assumed to vanish in non-convective areas in the grid box; this means $\bar{s} = \bar{s}_l$, $\bar{q}_{v+l} = \bar{q} = \bar{r}_v$. Then

$$\frac{\partial \bar{s}}{\partial t} = \left. \frac{\partial \bar{s}}{\partial t} \right|_{R.S.} - \frac{\partial}{\partial p} (\overline{\omega' s'_l}) + LC^* \quad (9.76)$$

where the subscript $R.S.$ denotes the resolved scale terms. Also

$$\frac{\partial \bar{q}}{\partial t} = \left. \frac{\partial \bar{q}}{\partial t} \right|_{R.S.} - \frac{\partial}{\partial p} (\overline{\omega' (q' + l')}) - C^* \quad (9.77)$$

The fluxes take the form:

$$F_{s_l} = -\frac{1}{g} \overline{(\omega' s'_l)} \approx -M_c(p) (\bar{s}(p) - s_c(p) + Ll(p)) \quad (9.78)$$

$$F_{q+l} = -\frac{1}{g} \overline{(\omega' (q' + l'))} \approx -M_c(p) (\bar{q} - q_c(p) - l(p)) \quad (9.79)$$

Hack's scheme deals directly with the finite difference partitioning in the vertical. It takes three adjacent levels into account as shown schematically in Fig. ??:

$$\frac{\partial \bar{s}_{k-1}}{\partial t} = \frac{g}{\Delta p_{k-1}} \{ \beta m_B (s_c - Ll_k - \bar{s}_{k-\frac{1}{2}}) \} \quad (9.80)$$

$$\begin{aligned} \frac{\partial \bar{s}_k}{\partial t} = \frac{g}{\Delta p_k} \{ & m_B (s_c - \bar{s}_{k+\frac{1}{2}}) \\ & - \beta m_B (s_c - Ll_k - \bar{s}_{k-\frac{1}{2}} + LR_k) \} \end{aligned} \quad (9.81)$$

$$\frac{\partial \bar{s}_{k+1}}{\partial t} = \frac{g}{\Delta p_{k+1}} \{ m_B (\bar{s}_{k+\frac{1}{2}} - s_c) \} \quad (9.82)$$

where $0 \leq \beta \leq 1$ is a detrainment parameter to be determined and m_B is the mass flux at the bottom of the condensation layer ($k + \frac{1}{2}$). Similar equations are written for $\partial \bar{q}_{k-1,k,k+1} / \partial t$. The rainout term above, $LR_k \equiv L(1 - \beta)m_B l_k$, now occurs instead of C^* . Assuming the large scale liquid water divergence is zero (i.e., no liquid water is stored in layer k), the rainout term can be calculated at level k using the saturation condition at level k . The liquid water into level $k - 1$, $\beta m_B Ll_k$, can now be computed. An equation for the moist static energy at level $k + 1$ is

$$\frac{\partial \bar{h}_{k+1}}{\partial t} = \frac{g}{\Delta p_{k+1}} m_B (\bar{h}_{k+\frac{1}{2}} - h_c) \approx \frac{\partial h_c}{\partial t} \quad (9.83)$$

The latter is the assumption that $\partial h' / \partial t$ can be neglected. An expression for the saturated moist energy, $\frac{\partial \bar{h}_k^*}{\partial t}$, can be derived leading to an expression

$$\frac{\partial (h_c - \bar{h}_k^*)}{\partial t} = \alpha m_B \quad (9.84)$$

where α consists of terms involving β , $\bar{h}_{k+\frac{1}{2}}$, \bar{s}_k , $\bar{s}_{k\pm\frac{1}{2}}$, etc. After this the mass flux can be written approximately as

$$m_B = \frac{(h_c - \bar{h}_k^*)}{\alpha \tau_{conv}} \quad (9.85)$$

where τ_{conv} is a characteristic convection time scale. Moist convection is possible anytime the numerator, $(h_c - \bar{h}_k^*) > 0$. The convective mass flux is then determined as a function of this stability measure. The remaining issue is to determine the detrainment parameter β . One constraint on its value is that the mass flux should be positive. Another is that the convection not supersaturate the detrainment layer, $k - 1$. Consistency of water and mass conservation lead to minimum and maximum values of β . Its actual value is chosen to be its minimum value added to a quantity proportional to the ratio of stability of the layers $k - 1$ and k .

In implementing the method, one starts at the boundary layer top and treats the next three layers above it. The procedure is repeated from the bottom of the free atmosphere to the top in steps of one layer at a time. Precipitation is calculated by adding the contributions from the layers.

$$P = \frac{1}{\rho_{H_2O}} \sum_{k=1}^K R_k \quad (9.86)$$

As with all convection schemes there are significant omissions in the Hack mass flux scheme. One is that the model assumes all the precipitation comes from the convection. In fact, a good portion of tropical precipitation comes from stratiform clouds adjacent to the convective cores. Another concern is the effect of downdrafts and the precipitation known to accompany them.

9.10.4 Improvements over CCM1

The CCM1 used a moist adiabatic adjustment (MAA) scheme. Hack (1994) has conducted an extensive comparison between the two implementations by using the CCM2 with both the simplified mass flux (SMF) formulation of the last subsection along with the MAA embedded in the same AGCM. A control run of 20 years of seasonal cycles with climatological sea surface temperatures utilizing the full CCM2 implementation with the SMF was compared with a 5 year run of the model with MAA embedded. The MAA model climate is both colder and dryer than the CCM2 model climate especially above the 850 mb level. In the tropics this leads to an improvement of over 10°C in the zonally averaged temperature at the 800 mb level. Not surprisingly the MAA atmosphere exhibits a different stability structure. The vertical heating profiles in the tropics are especially different with the MAA giving too much large scale heating concentrated along the trade inversion. In general the improvements

are dramatic for the CCM2 over the MAA formulation.

While the SMF seems to improve the climatological distribution of heating and water vapor in the tropical troposphere, it must be admitted that the model is still far from perfect. For example, the global average rainfall for January compares quite well with observations (analyses from the European Center for Medium Range Forecasting), the July average is about 30% too high, a feature common to both implementations. Most of the error seems to come from Southern Hemisphere midlatitudes, hence the cumulus parameterization may not be solely to blame. In spite of the attention given to cumulus parameterization in the last few years, it remains an unsolved problem and the source of much controversy in numerical experiments of climate change.

9.11 Gravity Wave Drag

When stably stratified air flows over corrugated terrain such as a series of parallel ridges, gravity waves are propagated with a component in the vertical. Since in the formulation of the equations of motion, the CCM2 explicitly filters out gravity waves some consideration must be given here, since the vertically propagating waves deliver horizontal momentum fluxes upwards. A vertical flux of horizontal momentum flux is a stress and the divergence (vertical derivative

here) of such a flux is effectively a frictional force on the layers of the flow. The CCM2 takes this effect into account in a very crude way based upon the linear theory of such wave excitation and propagation as explained in MacFarlane (1987) and in Lindzen (1993).

A brief description of the formulation based upon MacFarlane (1987) follows. As the air flows in the x direction over a topography, $Z = h \cos \mu x$, the stream function is assumed to be constant along this surface. Ignoring the Coriolis force we can proceed to write the equations of the flow introducing the stream function

$$w = \bar{U} \frac{\partial \psi}{\partial x} \quad (9.87)$$

$$u = -\frac{1}{\bar{\rho}} \frac{\partial}{\partial Z} (\bar{\rho} \bar{U} \psi) \quad (9.88)$$

$$\theta = -\psi \frac{\partial \bar{\theta}}{\partial Z} \quad (9.89)$$

and as mentioned above

$$\psi(x, 0) = h \cos \mu x \quad (9.90)$$

Making use of the equations of motion and the definitions above an equation for the stream function can be derived:

$$\frac{\partial}{\partial Z} \left[\frac{\bar{U}^2}{\bar{\rho}} \frac{\partial}{\partial Z} (\bar{\rho} \psi) \right] + N^2 \psi = 0 \quad (9.91)$$

where the Brunt-Väisälä frequency squared is defined by

$$N^2 = \frac{g}{\bar{\theta}} \frac{\partial \bar{\theta}}{\partial Z} \quad (9.92)$$

If the barred quantities vary sufficiently slowly in the vertical an approximate analytical solutions can be found

$$\psi(Z, x) = A(Z) \cos \left[\mu x + \int_0^Z \phi(Z') dZ' \right] \quad (9.93)$$

with

$$\phi(Z) = \frac{N}{\bar{U}} \quad (9.94)$$

$$A(Z) = h \left[\frac{\bar{\rho}(0)N(0)\bar{U}(0)}{\bar{\rho}N\bar{U}} \right]^{1/2} \quad (9.95)$$

Note that $A(Z)$ can get large as we go up since $\bar{\rho}$ is getting small and so can the other factors in the denominator. The mean vertical flux of horizontal momentum is

$$\tau = \frac{1}{L} \int_{-L/2}^{L/2} \bar{\rho} u w dx \quad (9.96)$$

where the length scale L of the integration covers at least one horizontal wavelength. Using the approximate solutions we obtain

$$\tau \approx -\frac{\mu h^2}{2} \bar{\rho}(0)N(0)\bar{U}(0) \quad (9.97)$$

which is independent of height. This means that when the conditions for this approximation are met, there will be no divergence of momentum flux and therefore no gravity wave drag.

The approximate solution obtained above and the inference about no drag clearly fail at so-called critical points where the amplitude $A(Z)$ becomes infinite (referred to as *wave breaking*). Even well away from these points the amplitude can become large enough for nonlinear effects to and saturation effects to become important. In particular, small scale instabilities might occur leading to convective overturning and dissipation of the wave. The condition for such an instability is expressed in terms of the local *Froude Number*

$$F(Z) = \frac{Nh}{\bar{U}} \left[\frac{\bar{\rho}(0)N(0)\bar{U}(0)}{\bar{\rho}N\bar{U}} \right]^{1/2} \quad (9.98)$$

which is a dimensionless measure similar to the Richardson Number. When it exceeds a critical value (unity here), the small scale convection begins to occur. Lindzen (1981) and Holton (1982) assumed that the net effect of the local instability is to cause turbulence which leads to local heat and momentum transports which eventually limit the amplitude of the wave. Hence, the local Froude number adjusts itself to never exceed a critical value. This process is called wave saturation.

The Froude number is generally below unity (stable) in the lower troposphere

although some exceptions occur very low in the Troposphere over terrain where nonlinear effects are important to the flow (see MacFarlane, 1987, for references). For the most part wave breaking phenomena are associated with the upper troposphere and levels above 30mb.

In CCM2 the wave momentum flux is assumed to be constant except in regions of wave saturation. The wave momentum flux is first computed in the lowest model level and continued upwards until a critical level is reached. In these regions the stress (momentum flux) at level $k + \frac{1}{2}$ is computed from

$$\tau_{k+1/2} = \min \left\{ \tau_{k+3/2}, \frac{E\mu F_c^2 \mathcal{U}_{k+1/2}^3 p_{k+1/2}^{n-1}}{2H_k N_{k+1/2}} \right\} \quad (9.99)$$

where E is an efficiency factor ($0 < E < 1$), μ is the wavenumber of the terrain generating the wave, F_c is the critical Froude number, taken in CCM2 to be $1/2$, \mathcal{U} is the component of the wind which is parallel to the reference level flow (near surface), $p_{k+1/2}^{n-1}$ is the pressure at the previous time step, and H_k is the local scale height. From adjacent layers a momentum flux divergence (friction force) can be calculated. In addition, a frictional heating term is added to the thermodynamic equation.

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